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# On the Continuous Phase Transition between Ferrielectric and Ferroelectric Smectics Induced by the Electric Field

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The phase transitions between ferrielectric phases nearby AF phase and ferroelectric one in the external field are studied by an extended ANNNI model, in which a temperature-dependent first neighbor interaction  $J_1 = ST - J_0$  is taken into account. Phase diagram of the external field vs.  $J_0$  exhibits the continuous phase transition from the ferrielectric phase to the ferroelectric one and seems to explain a new type of V-shaped response observed recently at ferrielectric phase in a certain smectic material.

**Keywords:** ferrielectric smectics; continuous phase transition; ANNNI model

## INTRODUCTION

In ferroelectric liquid crystals, it is well-known that between  $\text{SmC}^*$  and  $\text{SmC}_A^*$ , various kinds of phases;  $\text{SmC}^*$ , FI, AF,  $\text{FI}_H$ ,  $\text{SmC}_\gamma^*$ ,  $\text{FI}_L$  and  $\text{SmC}_A^*$ , appear in order as the temperature decreases<sup>[1~3]</sup>. Although not all of those phases appear in every materials, it is remarkable that several chiral smectic materials undergo such successive phase transitions between  $\text{SmC}^*$  and  $\text{SmC}_A^*$  phases. The mechanism of such successive

phase transitions have been explained by an extended ANNNI model<sup>[4~6]</sup> (hereafter, refer to E-ANNNI model), in which the third neighbor interaction  $J_3$  is added to ANNNI model<sup>[7,8]</sup> composed of the first neighbor one  $J_1$  and the second neighbor one  $J_2$  being negative.

In experiments of the ferroelectric or antiferroelectric smectic materials in the electric field, several interesting phenomena have been found<sup>[9,10]</sup>. In order to clarify those phenomena, we studied the phase diagram of the electric field vs.  $J_1$  by the use of the E-ANNNI model with the electric field<sup>[11,12]</sup>. From analysis of the phase diagram, we suggested that the phases  $FI_L$  and  $FI_H$  are of the structures with the wave numbers  $q = 2/5$  and  $2/7$  in respectively. Furthermore, for the temperature, corresponding to nearby the critical temperature without electric field, it was shown that the transition of second order occurs, especially the continuous transitions from mesophases with  $q < 1/4$  (the phase of  $q = 1/4$  is AF one) to ferroelectric one occur at the critical temperature. Recently, a new type of V-shaped response of the transmittance of light in the electric field is observed at a ferroelectric phase in a certain binary mixture of smectic material<sup>[13,14]</sup>. That phase seems to exhibit the continuous transition obtained here.

In this paper, we focus the modulated phases near AF phase in the electric field and study the continuous transition from these phases to the uniform phase in detail.

## E-ANNNI MODEL

One expects that  $J_1$  increases as temperature increases because of the increase of excluded volume of molecules with the temperature. Then, instead of  $J_1$  we use  $J_1(T)$  as the first neighbor interaction depending on temperature  $T$ , defined by  $J_1(T) = ST - J_0$ <sup>[4,5]</sup>, where  $J_0$  must be positive so that  $SmC_A^*$  phase is stable in the low temperature region.

In the mean field approximation, the free energy of the E-ANNNI

model with the external field  $E$  per molecule is given as<sup>[11,12]</sup>

$$F_p = \frac{1}{p} \sum_{i=1}^p \left[ -\frac{zJ}{2} \sigma_i^2 - J_1(T) \sigma_i \sigma_{i+1} - J_2 \sigma_i \sigma_{i+2} - J_3 \sigma_i \sigma_{i+3} - E \sigma_i + \frac{k_B T}{2} \{ (1 + \sigma_i) \ln(1 + \sigma_i) + (1 - \sigma_i) \ln(1 - \sigma_i) \} \right], \quad (1)$$

where  $p$  denotes the period of a ordered phase for the axial direction,  $J$  the interaction between molecules in a same layer and  $z$  the coordination number in a layer. The variable  $\sigma_i$  ( $= \sigma_{i+np}$  with integer  $n$ ) represents the order parameter which is determined by minimizing the free energy  $F_p$  with respect to  $\sigma_i$ , that is, by

$$E = -zJ\sigma_i - J_1(T)(\sigma_{i-1} + \sigma_{i+1}) - J_2(\sigma_{i-2} + \sigma_{i+2}) - J_3(\sigma_{i-3} + \sigma_{i+3}) + \frac{k_B T}{2} \ln \left\{ \frac{1 + \sigma_i}{1 - \sigma_i} \right\} \quad (i = 1, 2, 3, \dots, p). \quad (2)$$

On the other hand, in the case of continuous transition, transition point is determined by an instability point of the uniform phase. The condition of the instability of the uniform phase is given by<sup>[12]</sup>

$$\frac{k_B T}{1 - \sigma_0^2} = \alpha_p(q_m), \quad (3)$$

where  $\sigma_0$  means a order parameter of the uniform phase on the instability surface  $E = J_0 = T$ . The  $\alpha_p(q_m)$  expresses the maximum value of

$$\alpha_p(q) = zJ + 2\{J_1(T) \cos(2\pi q) + J_2 \cos(4\pi q) + J_3 \cos(6\pi q)\} \quad (4)$$

with respect to  $q$ , where  $q$  is the wave number of the modulated phase. The phase with the wave number  $q_m$  is just one appearing at the transition point.

## PHASE DIAGRAM

Here, all energy parameters are scaled in the unit  $|J_2|$ , that is  $|J_2| = 1$ , and  $J = 1$ ,  $J_2 = -1$  and  $J_3 = 0.3$  are adopted, additionally we choose  $z = 6$  and  $S = 2$ . The phase diagram of  $T$  vs.  $J_0$  for  $E = 0$  near AF phase

( $q = 1/4$ ) is shown in Fig.1, where phases are indicated by the values of  $q$ . The dotted line, which is obtained from eq.(3) with  $\sigma_0 = 0$ , shows the critical temperature taking a minimum value at  $T = 8$ . It is noted that the uniform phase  $\text{SmC}^*$  ( $q = 0$ ) appear at high temperature side of disordered phase when  $S > 1/2$ .

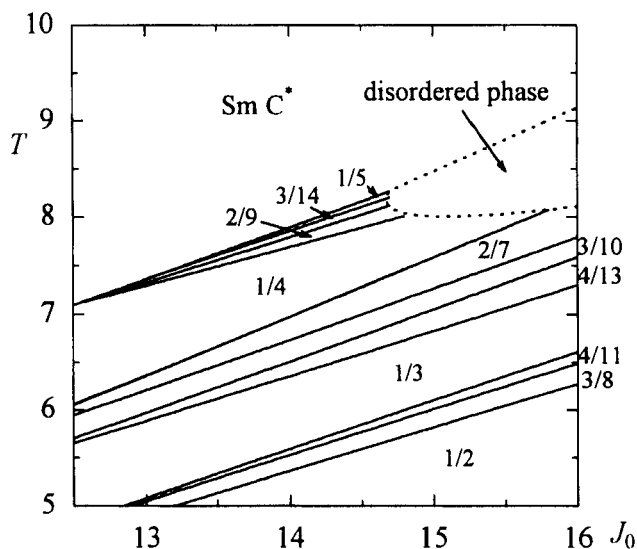
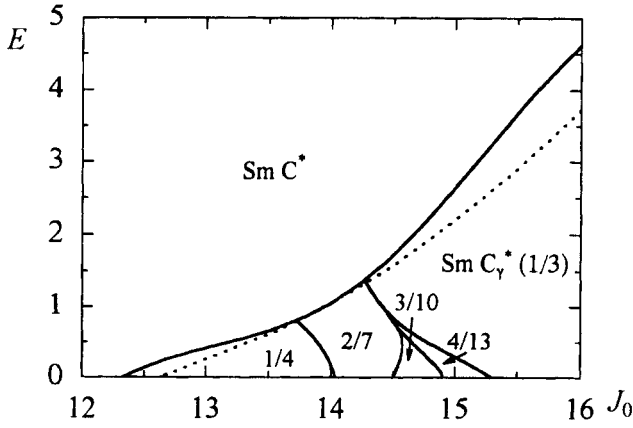
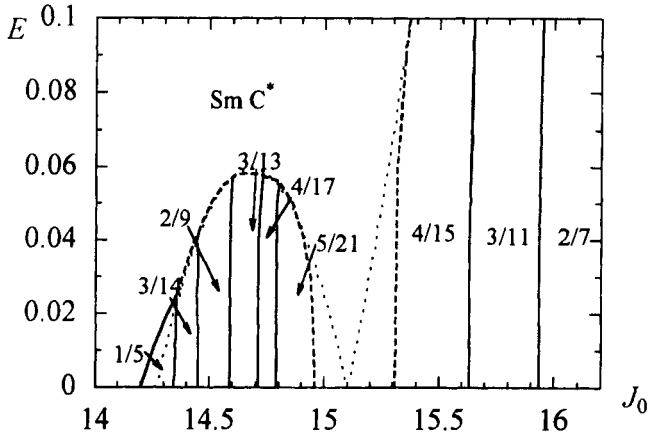


FIGURE 1 : Phase diagram  $T$  vs.  $J_0$  at  $E = 0$ .

The phase diagrams of  $E$  vs.  $J_0$  near  $q = 1/4$  are shown in Fig.2a at  $T = 7$  and in Fig.2b at  $T = 8$ , respectively. The solid and the broken lines represent the coexisting and the critical lines, respectively, the dotted line the instability line obtained from eq.(3). The phase diagram at  $T = 7$  shows that the transition from modulated phase to uniform phase is the first order.

FIGURE 2a : Phase diagram  $E$  vs.  $J_0$  for  $T = 7$ .FIGURE 2b : Phase diagram  $E$  vs.  $J_0$  for  $T = 8$ .

On the other hand, at  $T = 8$ , we have shown previously<sup>[12]</sup> that the continuous phase transition from the modulated phase to the uniform one occurs near the phase  $q = 1/4$ , in which phases with  $q = 1/5, 3/14, 2/9, 3/13$  and  $2/7$  are taken into account. In order to study the phase

diagram between the phases  $q = 3/13$  and  $2/7$  precisely, here phases with  $q = 4/17, 5/21, 4/15$  and  $3/11$  are taken into account in addition to those mentioned above. The critical line for the modulated phases with  $q < 2/7$  agrees approximately with the instability line except for the phases with  $q = 5/21$  and  $4/15$ , which are adjacent to the phase  $q = 1/4$ . If this analysis is continued by taking account of more phases between  $q = 5/21$  and  $4/15$ , the critical line seems to agree completely with the instability line in the interval between  $q = 3/14$  and  $2/7$ . Thus, we are led to the conclusion that the second order transition truly occurs and the transition temperature is given by the instability condition of eq.(3) with continuous wave number  $q_m$ .

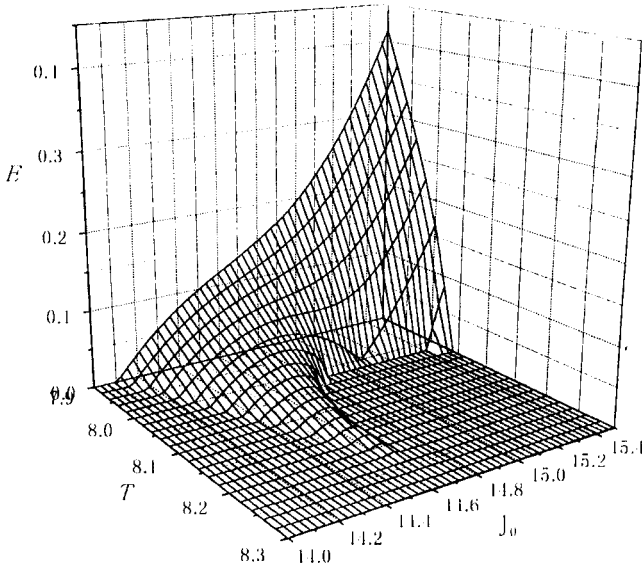


FIGURE 3 : Phase diagram  $E(J_0, T)$

Consequently, we conclude that the phase diagram nearby  $T = 8$  with  $q < 2/7$  is replaced by the critical surface  $E(J_0, T)$  obtained from eq.(3), below of which modulated (incommensurate) phases with continuous wave



number  $q_m$  become stable. The phase diagram  $E(J_0, T)$  near  $T = 8$  is shown in Fig.3, in Fig.4, the modulated phases with the typical wave number  $q_m$  are shown. It is noted that the region of the modulated phases with  $q < 1/4$  disappear above  $T \simeq 8.26$ .

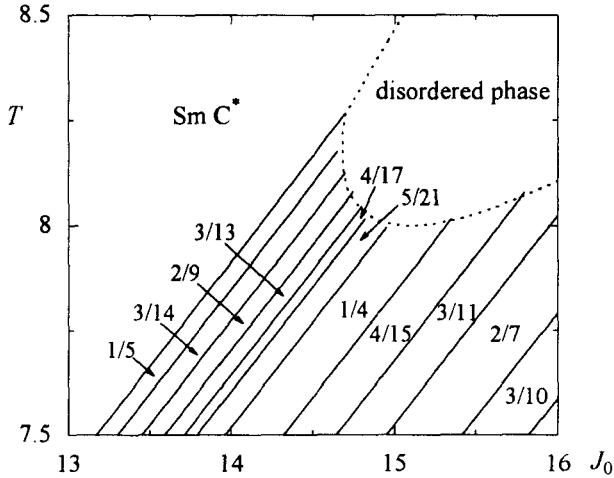


FIGURE 4 : Modulated phases  $q_m$  on the critical surface.

## SUMMARY

The phase diagram  $E(J_0, T)$  near the critical temperature of AF phase is obtained, where the phase transition between the uniform phase and the modulated one is continuous and the modulated phase changes continuously to some extent in the area near the AF phase.

As mentioned above, recently the V-shaped response of the transmittance of light to the external field at the antiferroelectric smectic materials in the planer texture is reported<sup>[14]</sup> and such response is assumed to come from some kind of randomness of the layers. However the sharpness of the response curve suggests the existence of a long range order and the response without hysteresis should be attributed to the continuous phase transition. While, at the free standing film of the same

material, successive phase transition is observed, in which jumps of the helical pitch are observed in the ferroelectric phase with the change of temperature<sup>[14]</sup>. The phases of this situation are considered to be far from the critical temperature and located at the region that the devil's staircase appear. On the other hand, in the former case of the homogeneous sample, the order is suppressed by the walls and the sequence of phases is not observed. In the present stage, a structure of the phase named as SmX\* characterized by the V-shaped response is not clear, the phase is supposed to correspond to the one near the critical temperature where the phase changes continuously, provided that some long range order exists.

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